

Distributions

Name	pdf	mean	variance
Hypergeometric	$p(x) = \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}}$ $x=0,1,2,\dots,\min(n,D)$	$\frac{nD}{N}$	$\frac{nD}{N} \left(1 - \frac{D}{N}\right) \left(\frac{N-n}{N-1}\right)$
Binomial	$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ $x = 0,1,2,\dots,n$	np	$np(1-p)$
Poisson	$p(x) = \frac{e^{-\lambda} \lambda^x}{x!}$ $x = 0,1,\dots$	λ	λ
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty$	μ	σ^2
Exponential	$f(x) = \lambda e^{-\lambda x}$ $x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Two-sided confidence intervals

<i>statistic</i>	<i>Interval</i>
mean (variance known)	$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
mean (variance unknown)	$\bar{x} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}$
variance	$\frac{(n-1)S^2}{X_{\alpha/2, n-1}^2} \leq \sigma^2 \leq \frac{(n-1)S^2}{X_{1-\alpha/2, n-1}^2}$
difference between means (variance known)	$\bar{x}_1 - \bar{x}_2 - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{x}_1 - \bar{x}_2 + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
difference between means (variance unknown)	$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{x}_1 - \bar{x}_2 + t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ where $S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$
Ratio of variances (for normal distributions)	$\frac{S_1^2}{S_2^2} F_{1-\alpha/2, n_2-1, n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} F_{\alpha/2, n_2-1, n_1-1}$

Hypothesis tests:

Test on means with known variance		
Hypothesis	Criteria for Rejection	Test Statistic
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$ Z_0 > Z_{\alpha/2}$	
$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$Z_0 < -Z_\alpha$	$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	$Z_0 > Z_\alpha$	
$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 \neq \mu_2$	$ Z_0 > Z_{\alpha/2}$	
$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 < \mu_2$	$Z_0 < -Z_\alpha$	$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 > \mu_2$	$Z_0 > Z_\alpha$	

Hypothesis tests (continued):

Test on means of normal distributions with unknown variance		
Hypothesis	Criteria for Rejection	Test Statistic
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$ t_0 > t_{\alpha/2, n-1}$	
$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$
$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	
$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 \neq \mu_2$	$ t_0 > t_{\alpha/2, v}$	<p style="text-align: center;">if $\sigma_1^2 = \sigma_2^2$,</p> $t_0 = \frac{\bar{x}_1 - \bar{x}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ $v = n_1 + n_2 - 2$
$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 < \mu_2$	$t_0 < -t_{\alpha, v}$	<p style="text-align: center;">if $\sigma_1^2 \neq \sigma_2^2$,</p> $t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$ $v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right)^2}{\frac{(S_1^2/n_1)^2}{n_1+1} + \frac{(S_2^2/n_2)^2}{n_2+1}} - 2$
$H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 > \mu_2$	$t_0 > t_{\alpha, v}$	

Hypothesis tests (continued):

Tests on variances of normal distributions		
Hypothesis	Criteria for Rejection	Test Statistic
$H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 \neq \sigma_0^2$	$X_0^2 > X_{\alpha/2, n-1}^2 \text{ or}$ $X_0^2 < X_{1-\alpha/2, n-1}^2$	
$H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 < \sigma_0^2$	$X_0^2 < X_{1-\alpha, n-1}^2$	$X_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$
$H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 > \sigma_0^2$	$X_0^2 > X_{\alpha, n-1}^2$	
$H_0 : \sigma_1^2 = \sigma_2^2$ $H_1 : \sigma_1^2 \neq \sigma_2^2$	$F_0 > F_{\alpha/2, n_1-1, n_2-1} \text{ or}$ $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$	$F_0 = \frac{S_1^2}{S_2^2}$
$H_0 : \sigma_1^2 = \sigma_2^2$ $H_1 : \sigma_1^2 < \sigma_2^2$	$F_0 > F_{\alpha, n_2-1, n_1-1}$	$F_0 = \frac{S_2^2}{S_1^2}$
$H_0 : \sigma_1^2 = \sigma_2^2$ $H_1 : \sigma_1^2 > \sigma_2^2$	$F_0 > F_{\alpha, n_1-1, n_2-1}$	$F_0 = \frac{S_1^2}{S_2^2}$

Equation Sheet for MEGR 3282: SPC & Metrology

Variables control charts:

μ, σ given.		
Quality Characteristic	Center Line	Control Limits
\bar{x}	μ	$\mu \pm A\sigma$
R	$d_2\sigma$	$UCL = D_2\sigma, LCL = D_1\sigma$
S	$c_4\sigma$	$UCL = B_6\sigma, LCL = D_5\sigma$
μ, σ not given (unknown).		
\bar{x}	$\bar{\bar{x}}$	$\bar{\bar{x}} \pm A_2\bar{R}, \text{ or } \bar{\bar{x}} \pm A_3\bar{S}$
R	\bar{R}	$UCL = D_4\bar{R}, LCL = D_3\bar{R}$
S	\bar{S}	$UCL = B_4\bar{S}, LCL = B_3\bar{S}$

Attributes control charts:

Quality Characteristic	Center Line	Control Limits
Fraction non-conforming (p given)	p	$p \pm 3\sqrt{\frac{p(1-p)}{n}}$
Fraction non-conforming (p not given)	\bar{p}	$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
Number non-conforming (p given)	np	$np \pm 3\sqrt{np(1-p)}$
Number non-conforming (p not given)	$n\bar{p}$	$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$
Non-conformities (c given)	c	$c \pm 3\sqrt{c}$
Non-conformities (c not given)	\bar{c}	$\bar{c} \pm 3\sqrt{\bar{c}}$
Average number of non-conformities per unit	\bar{u}	$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{n}}$

Confidence Interval for PCR

$$P\hat{C}R \sqrt{\frac{X_{1-\alpha/2, n-1}^2}{n-1}} \leq PCR \leq P\hat{C}R \sqrt{\frac{X_{\alpha/2, n-1}^2}{n-1}}$$

Hypothesis Testing from PCR

Sample Size, n	$\alpha = \beta = 0.10$		$\alpha = \beta = 0.05$	
	$\frac{PCR(\text{high})}{PCR(\text{low})}$	$\frac{C}{PCR(\text{low})}$	$\frac{PCR(\text{high})}{PCR(\text{low})}$	$\frac{C}{PCR(\text{low})}$
10	1.88	1.27	2.26	1.37
20	1.53	1.20	1.73	1.26
30	1.41	1.16	1.55	1.21
40	1.34	1.14	1.46	1.18
50	1.30	1.13	1.40	1.16
60	1.27	1.11	1.36	1.15
70	1.25	1.10	1.33	1.14
80	1.23	1.10	1.30	1.13
90	1.21	1.10	1.28	1.12
100	1.20	1.09	1.26	1.11